Some Measures of Photographs of the Pleiades at the Oxford University Observatory. By Professor H. H. Turner, M.A., B.Sc.

1. The late Professor Pritchard directed his assistants to take a number of photographs of the *Pleiades* with the Astrographic Equatorial, to be accurately measured with the micrometer. His original intention was to discuss the results for relative parallax of the individual stars; while he expected at the same time to gain experience in the measurement and reduction of stellar photographs taken with this instrument, for use in discussing the plates of the Astrographic Chart. Various difficulties arose in the course of the work, which made him doubtful of being able to carry out satisfactorily his original schemes; but he expressed the hope that "at all events the attempt would issue in a catalogue of the coordinates of all the stars therein up to magnitude 12" (*Monthly Notices*, vol. liii. p. 249).

2. On being appointed to succeed Professor Pritchard, it was one of my first cares to examine the actual state of this work, the latest initiated by him. The Radcliffe Observer, who had kindly undertaken the supervision of the work of the University Observatory in the interval between the death of Professor Pritchard and the appointment of a successor, had very naturally directed that this work should be continued without change on the lines laid down by my illustrious predecessor.

3. The following is a brief statement of the position of the work as I found it at the end of 1893 December.

Seventy plates of the *Pleiades* group had been taken between 1892 August 11 and 1893 March 4.

Three plates had been completely measured in a micrometer constructed by Messrs. Troughton and Simms, furnished with a micrometer screw (in one coordinate only) reading to $0^{\mathrm{mm}}\cdot01$ ($0^{\mathrm{mm}}\cdot001$ by estimation): all stars visible within a radius of 40′ from *Alcyone* having been measured by reference to the *réseau* lines: viz.—

Plate 252, exposed for 6^{m} on 1892 December 12;
" 259, " 7^{m} " " " 13;
" 294, " 2^{m} " 1893 February 10;

also a few of the brightest stars on plates 120, 121, 128, 129, 151, 152, 153, and 258, but the measures of these eight plates were made with a different micrometer furnished with ten wires.

Reductions.—The focal length of the telescope (or scale in arc of the photograph) had been discussed for eight plates; the orientation for nine plates; and the rectilinear measures of a few stars had been corrected for refraction, precession, aberration and

nutation, converted into differences of R.A. and Decl. from *Alcyone*, and compared with the corresponding differences found by Elkin with the heliometer in 1885, reduced to the epoch 1892·0.

But as not more than ten to fifteen stars had been measured, except on plates 252, 259, and 294, I thought it desirable to discuss these three plates first, before having time spent upon the completion of the measurement of the other plates.

4. It further appeared that this work had occupied considerable time; and that, if the original intention of Professor Pritchard were carried out completely, the pressing work of dealing in some way or other with the accumulated photographs of the Astrographic Chart must be delayed for some years.

5. I therefore decided to terminate this work (which I believe Professor Pritchard only meant to be preliminary to that of dealing with the Chart photographs) as speedily as possible, so far as was consistent with making a proper use of the time already expended. The method of reduction of the plates, by differences of R.A. and Decl., which had been found very tedious and not free from pitfalls, was at once changed for that of discussion in rectilinear coordinates such as I have recently suggested (*Monthly Notices*, vol. liv. p. 11). As a consequence, the reductions of the three plates which had been more fully measured were rapidly completed, and the results which follow will, I hope, be found of value from three points of view:

(1) As throwing further light on the accuracy to be reasonably expected from measures of stellar photographs.

(2) As giving a series of relative positions of the *Pleiades* at the epoch in question.

(3) As illustrating some advantages of the method of discussion by rectilinear coordinates, which I think have not hitherto been sufficiently recognised.

6. The method of reduction adopted may be briefly explained as follows. The positions of the stars given by Elkin for 1885·0* were brought up to 1892·0, and were used to find the quantities

$$\xi = \frac{180 \times 60}{\pi} \cdot \frac{\tan(\alpha - A)\sin q}{\cos(P - q)}, \qquad \eta = \frac{180 \times 60}{\pi} \cdot \tan(P - q);$$

where

A, P are the R.A. and N.P.D. of *Alcyone*,

α, p ,, ,, ,, of any other star,

and $\tan q = \tan p \cdot \cos(\alpha - A)$.

[In practice these formulæ may be simplified into approxi-

* *Transactions of the Yale University Observatory*, vol. i. part 1, pp. 86 and 87.

mate formulæ; but I give them in their geometrical form, as quoted in my paper above referred to.]

ξ and η are thus the rectangular coordinates which any star would have with reference to *Alcyone* on a photographic plate free from all errors, with *Alcyone* at its centre; the coordinates being measured parallel and perpendicular to the equator respectively, and expressed in minutes of arc. Our actual plate is affected by refraction and aberration; the coordinates are measured only approximately in the proper directions, and we only know the scale value approximately. Further, there are accidental deviations from flatness in the plate, and optical and photographic distortions. These last we must neglect as accidental, but all the former we may consider sensibly allowed for by linear corrections. If x, y be the measured coordinates of a star in millimetres, and affected with the above sources of error, then

$$x-\xi=a\xi+b\eta+c$$
$$y-\eta=d\xi+e\eta+f,$$

where a, b, c, d, e, f are six constants for the plate, whose values are small. Theoretically the six measured coordinates of three stars are sufficient to give these six constants; but practically it is better to use many or all the measures, and solve the resulting equations by least squares or an equivalent process.

7. Thus the places given by Elkin having once been converted into ξs and ηs, all we have to do is to form the differences $x-\xi$, $y-\eta$ for the stars on any plate, and solve a series of equations of the above form. It will be remarked that the coefficients of a, b, c, d, e, f on the right of the equations are ξ, η, 1, and are thus the same for all plates and for both sets of equations. Much of the arithmetic in this particular case of measuring several plates of the same region can therefore be saved. This advantage is gained by starting with the *given* places (Elkin's) and reducing them to compare with the measured. It disappears when we start with the measured places (varying for each plate) and work back to the given places.

8. Having got a, b, c, d, e, f, the formation of the quantities

$$r_x \equiv x-\xi-(a\xi+b\eta+c)$$
$$r_y \equiv y-\eta-(d\xi+e\eta+f)$$

is very simple, and r_x, r_y then represent proper motion + accidental error for any star. If stars not given by Elkin are measured on the plate, we can deduce their positions (ξ, η) as they would be on his system from the equations

$$x=(1+a)\xi+b\eta+c$$
$$y=d\xi+(1+e)\eta+f,$$

or rather their equivalents

$$\Delta \, . \, \xi = (1+e)\,x - by - c\,(1+e) + bf$$
$$\Delta \, . \, \eta = -dx + (1+a)\,y - f\,(1+a) + cd,$$

where

$$\Delta = 1 + a + e + ac - bd,$$

which, if a, b, c, &c., are small, become sensibly

$$\xi = (1-a)\,x - by - c$$
$$\eta = -dx + (1-e)\,y - f.$$

9. In the following table (Table I.) are given :

Column 1. The star's number in Elkin's paper above referred to (*Transactions of the Yale University Observatory*, vol. i. part 1, pp. 86 and 87).

Columns 2 and 6. The coordinates ξ and η, representing the rectangular coordinates of a star on a perfect plate with *Alcyone* as centre, according to Elkin's places.

Columns 3 to 5. The simple differences between the measured coordinates in the ξ direction on each plate, and those on a hypothetically perfect plate as above.

Columns 7 to 9. The similar differences for the η coordinates.

TABLE I.

No. in Elkin.	ξ 1892·0.	$x-\xi$ Plate 252.	$x-\xi$ Plate 259.	$x-\xi$ Plate 294.	η 1892·0.	$y-\eta$ Plate 252.	$y-\eta$ Plate 259.	$y-\eta$ Plate 294.
	′	′	′	′	′	′	′	′
3	−42·345	−0·326	−0·340	−0·330	+ 1·363	−0·003	−0·012	+0·080
5	−36·760	−0·276	−0·303	−0·270	+10·803	+0·062	+0·052	+0·136
6	−35·722	−0·271	−0·291	−0·290	+ 0·238	−0·016	+0·004	+0·059
7	−32·578	−0·239	−0·276	−0·235	+ 9·263	+0·052	+0·044	+0·135
8	−32·684	−0·252	−0·236	−0·295	−24·389	−0·202	−0·212	−0·126
10	−31·286	−0·235	−0·275	−0·209	+21·495	+0·168	+0·160	+0·219
13	−28·029	−0·212	−0·217	−0·221	− 4·413	−0·046	−0·056	+0·017
14	−26·290	−0·194	−0·227	−0·163	+21·287	+0·154	+0·145	+0·208
15	−25·931	−0·190	−0·211	−0·197	− 1·515	−0·031	−0·029	+0·028
17	−25·480	−0·184	−0·218	−0·178	+13·629	+0·092	+0·090	+0·147
18	−25·053	−0·179	−0·223	−0·135	+31·132	+0·236	+0·223	+0·277
19	−24·645	−0·184	−0·203	−0·164	+10·817	+0·073	+0·061	+0·123
20	−22·804	−0·183	−0·199	−0·155	+15·574	+0·132	+0·105	+0·168
21	−22·208	−0·169	−0·172	−0·175	− 4·157	−0·042	−0·049	+0·002
22	−21·768	−0·161	−0·198	−0·137	+26·789	+0·197	+0·196	+0·233
23	−19·838	−0·143	−0·179	−0·116	+25·202	+0·186	+0·173	+0·224
24	−17·222	−0·125	−0·154	−0·126	+ 5·276	+0·030	+0·035	+0·070
25	−16·710	−0·129	−0·149	−0·125	+ 4·942	+0·022	+0·030	+0·063
26	−15·788	−0·136	−0·119	−0·140	− 9·539	−0·081	−0·074	−0·062

No. in Elkin.	ξ 1892·0.	x−ξ Plate 252.	x−ξ Plate 259.	x−ξ Plate 294.	η 1892·0.	y−η Plate 252.	y−η Plate 259.	y−η Plate 294.
	′	′	′	′	′	′	′	′
27	−14·173	−0·098	−0·122	−0·096	+ 8·869	+0·058	+0·055	+0·090
28	−12·298	−0·090	−0·072	−0·131	−28·971	−0·242	−0·242	−0·210
29	−11·348	−0·081	−0·107	−0·085	− 0·209	−0·014	−0·021	+0·010
30	− 6·986	−0·041	−0·096	−0·018	+24·840	+0·181	+0·187	+0·198
32	− 5·617	−0·038	−0·057	−0·052	− 6·640	−0·072	−0·060	−0·055
33	− 4·432	−0·025	−0·060	−0·012	+10·251	+0·076	+0·079	+0·083
34	− 4·140	−0·037	−0·033	−0·066	−19·493	−0·161	−0·149	−0·145
35	− 2·868	−0·025	−0·027	−0·025	+ 1·363	+0·001	+0·014	+0·017
36	− 2·411	−0·028	−0·017	−0·060	−22·760	−0·199	−0·182	−0·174
37	− 2·228	−0·028	−0·036	−0·032	+ 2·015	−0·003	+0·020	+0·017
38	− 1·840	−0·016	−0·017	−0·019	+ 0·648	+0·001	+0·011	+0·005
39	− 1·682	−0·012	−0·008	−0·032	−18·122	−0·152	−0·136	−0·150
40	− 1·420	+0·005	−0·047	+0·042	+28·989	+0·206	+0·228	+0·225
41	− 0·857	−0·008	−0·051	+0·046	+33·122	+0·247	+0·274	+0·261
42	− 0·956	+0·030	−0·006	−0·023	−11·439	−0·099	−0·076	−0·092
43	− 0·275	+0·001	+0·013	−0·038	−25·611	−0·209	−0·193	−0·201
44	− 0·017	+0·007	−0·014	+0·015	+10·995	+0·076	+0·085	+0·082
46	+ 2·016	+0·030	+0·044	−0·022	−29·718	−0·248	−0·225	−0·236
47	+ 3·349	+0·014	+0·042	−0·026	−33·687	−0·281	−0·251	−0·271

No in Elkin.	ξ 1892·0.	$x-\xi$ Plate 252.	$x-\xi$ Plate 259.	$x-\xi$ Plate 294.	η 1892·0.	$y-\eta$ Plate 252.	$y-\eta$ Plate 259.	$y-\eta$ Plate 294.
	′	′	′	′	′	′	′	′
49	+10·010	+0·084	+0·059	+0·101	+12·899	+0·090	+0·113	+0·085
50	+ 8·667	+0·077	+0·064	+0·076	+ 2·300	+0·006	+0·037	+0·005
51	+12·260	+0·092	+0·113	+0·035	−40·910	−0·325	−0·300	−0·335
52	+13·763	+0·110	+0·082	+0·131	+14·562	+0·104	+0·132	+0·094
53	+14·944	+0·121	+0·103	+0·127	+ 4·717	+0·032	+0·049	+0·010
54	+20·174	+0·166	+0·162	+0·138	−14·638	−0·125	−0·098	−0·153
55	+23·010	+0·181	+0·171	+0·171	− 2·850	−0·032	+0·010	−0·057
56	+23·281	+0·187	+0·180	+0·187	+ 2·159	+0·004	+0·048	−0·041
57	+23·693	+0·187	+0·186	+0·163	−12·838	−0·114	−0·075	−0·144
58	+24·356	+0·196	+0·172	+0·221	+17·733	+0·129	+0·164	+0·093
59	+25·497	+0·206	+0·176	+0·225	+16·837	+0·118	+0·155	+0·093
60	+26·556	+0·223	+0·200	+0·226	+ 8·854	+0·058	+0·092	+0·024
61	+27·984	+0·218	+0·173	...	+31·728	+0·231	+0·279	...
62	+31·004	+0·241	+0·249	+0·200	−23·239	−0·191	−0·151	−0·230
63	+31·158	+0·244	+0·210	+0·260	+ 8·714	+0·063	+0·108	+0·023
64	+33·157	+0·259	+0·246	+0·277	+ 7·103	+0·058	+0·109	+0·016
65	+33·460	+0·254	+0·242	+0·288	+15·025	+0·104	+0·152	+0·064
66	+34·208	+0·273	+0·255	+0·246	−14·986	−0·127	−0·072	−0·173

Plate 294, No. 61, the image excessively faint.

10. To find the coefficients *a*, *b*, *c*, *d*, *e*, *f*, the following four groups of stars were selected :

Nos.	3, 5, 6, 7, 13, 15, 21	=	Group I.
„	30, 40, 41	=	„ II.
„	34, 43, 46, 47	=	„ III.
„	55, 56, 60, 63, 64	=	„ IV.

The mean coordinates of each group are roughly as follows :

Group	I.	−30′	0′
„	II.	0	+30
„	III.	0	−30
„	IV.	+30	0

the four resulting equations being thus well adapted to the determination of the quantities *a*, *b*, *c*, *d*, *e*, *f*.

11. Although scarcely necessary, an example may perhaps be given of the formation of one pair of the equations ; say that for Plate 252, Group I. :

No. in Elkin.	ξ	$x-\xi$	η	$y-\eta$
3	−42′345	−0′326	+ 1′363	−0′003
5	−36·760	−0·276	+10·803	+0·062
6	−35·722	−0·271	+ 0·238	−0·016
7	−32·578	−0·239	+ 9·263	+0·052
13	−28·029	−0·212	− 4·413	−0·046
15	−25·931	−0·190	− 1·515	−0·031
21	−22·208	−0·169	− 4·157	−0·042
Mean	−31·9390	−0·2404	+ 1·6546	−0·0034

The resulting equations are thus—

$$-31{\cdot}9390a + 1{\cdot}6546b + c = -0'{\cdot}2404$$
$$-31{\cdot}9390d + 1{\cdot}6546e + f = -0{\cdot}0034$$

For the same group in other plates, columns 2 and 4 need not be repeated.

12. The following Table II. gives a summary of the numbers entering into the equations. The coefficients of *a* and *b* (or of *d* and *e*) are given in the second and third columns ; the means of $x-\xi$ in columns 4 to 6, and the means of $y-\eta$ in columns 7 to 9.

TABLE II.

Group.	Coefficient of a or d.	Coefficient of b or e.	Mean $x-\xi$. Pl. 252.	Pl. 259.	Pl. 294.	Mean $y-\eta$. Pl. 252.	Pl. 259.	Pl. 294.
I.	−31·9390	+ 1·6546	−0·2404	−0·2586	−0·2454	−0·0034	−0·0069	+0·0653
II.	− 3·0877	+28·9837	−0·0147	−0·0647	+0·0233	+0·2113	+0·2297	+0·2280
III.	+ 0·2375	−27·1273	+0·0020	+0·0165	−0·0380	−0·2247	−0·2045	−0·2132
IV.	+27·4324	+ 4·7960	+0·2188	+0·2014	+0·2242	+0·0302	+0·0734	−0·0070

13. The resulting values of a, b, c, d, e, f are given in Table III. below :

TABLE III.

Plate.	a	b	c	d	e	f
252	+0·00773	+0·00016	−0·00535	+0·00015	+0·00778	+0·01257
259	+0·00780	−0·00099	+0·00992	+0·00094	+0·00780	−0·00838
294	+0·00783	+0·00155	−0·00218	−0·00163	+0·00777	+0·00090

14. Substituting these values in the expressions

$$r_x \equiv x - \xi - (a\xi + b\eta + c)$$
$$r_y \equiv y - \eta - (d\xi + e\eta + f)$$

we get the values for r_x and r_y, shown in Table IV. ; the values being shown in seconds of arc as a more familiar unit for small quantities.

TABLE IV.

Name or No. in Elkin.		Plate 252 r_x	Plate 259 r_x	Plate 294 r_x	Mean.	Plate 252 r_y	Plate 259 r_y	Plate 294 r_y	Mean.
	3	+0″·21	−0″·02	+0″·08	+0″·09	−0″·24	−0″·69	−0″·12	−0″·35
Celæno	5	−0·12	−0·23	−0·02	−0·12	+0·34	+0·21	+0·31	+0·29
Electra	6	−0·04	+0·19	+0·70	−0·28	+0·10	−1·79	−0·05	−0·58
	7	−0·41	+0·20	−0·29	−0·17	+0·24	+0·21	−0·69	−0·08
	8	−0·02	−0·23	+0·13	−0·04	−0·20	−0·19	−0·72	−0·37
Taygeta	10	+0·04	+0·02	−0·10	−0·01	−1·00	−0·95	−0·15	−0·70
	13	−0·05	−0·37	−0·29	−0·24	−0·20	+0·07	−0·44	−0·19
	14	−0·10	−0·51	−0·53	−0·38	−0·19	+0·14	−0·08	−0·04
	15	−0·36	+0·08	−0·45	−0·24	+0·27	−0·08	+0·05	+0·08
	17	−0·40	−0·22	+0·03	−0·20	−0·06	−0·07	−0·08	−0·07
	18	−0·34	−0·78	−0·73	−0·62	−0·52	+0·11	+0·23	−0·06
	19	−0·02	−0·52	−0·68	−0·41	−0·20	+0·42	−0·04	+0·06
Maia	20	+0·83	−0·21	+0·10	+0·24	−1·50	+0·07	−0·68	−0·70
	21	+0·05	−0·37	−0·26	−0·19	−0·27	+0·10	+0·02	−0·05
Asterope k	22	+0·08	−0·43	+0·54	+0·06	−0·15	−0·09	+0·55	+0·10
„ l	23	−0·11	−0·58	+0·04	−0·22	−0·22	+0·65	+0·16	+0·20
	24	−0·18	+0·32	+0·02	+0·05	−0·15	−0·25	−0·15	−0·18
	25	+0·29	+0·28	+0·17	+0·25	−0·18	−0·08	+0·07	−0·06
Merope	26	+1·00	−0·23	+0·15	−0·31	−0·38	−0·56	+0·72	−0·07
	27	−0·38	−0·41	−0·02	−0·27	−0·12	+0·41	+0·03	+0·11
	28	−0·32	−0·24	−0·56	−0·37	+0·23	+0·62	+0·20	+0·35
	29	−0·12	+0·60	−0·19	+0·10	−0·01	+0·88	+0·32	+0·40
	30	−0·28	+0·46	+0·16	+0·11	+0·02	+0·34	+0·27	+0·21
	32	−0·13	+0·64	−0·08	+0·14	+0·50	+0·53	+0·65	+0·56

Name or No. in Elkin.	Plate 252 r_x	Plate 259 r_x	Plate 294 r_x	Mean.	Plate 252 r_y	Plate 259 r_y	Plate 294 r_y	Mean.
	″	″	″	″	″	″	″	″
33	−0·25	+0·37	−0·35	−0·08	−0·47	+0·18	+0·13	−0·05
34	+0·37	+0·67	+0·26	+0·43	−0·15	−0·07	−0·07	−0·10
35	+0·43	−0·35	+0·33	+0·14	−0·10	−0·01	−0·20	−0·10
36	+0·60	+0·71	+0·40	+0·57	+0·62	+0·47	−0·04	+0·35
37	+0·91	+0·45	+1·11	+0·82	+0·45	−0·02	+0·04	+0·16
38	+0·37	−0·42	+0·39	+0·11	−0·43	−0·10	+0·08	−0·15
39	+0·02	+0·23	−0·50	−0·08	0·00	−0·05	+0·64	+0·20
40	−0·43	−0·13	−0·43	−0·33	+0·51	+0·16	+0·04	+0·24
41	+0·65	+0·13	−0·03	+0·25	−0·03	−0·64	−0·24	−0·30
42	−0·48	+0·05	−0·08	−0·17	−0·05	−0·47	+0·20	−0·11
43	−0·18	+0·08	−0·18	−0·09	−0·07	−0·07	+0·05	−0·03
44	−0·07	−0·38	+0·17	−0·09	−0·08	+0·40	+0·11	+0·15
Alcyone	+0·25	−0·55	+0·05	−0·08	−0·65	+0·35	−0·10	−0·05
46	−0·90	−0·46	−0·44	−0·60	+0·38	+0·07	+0·02	+0·16
47	+0·64	−0·08	+0·05	+0·20	+0·52	−0·17	+0·13	+0·16
49	−0·02	−0·17	−0·10	−0·10	+0·07	+0·17	−0·16	+0·03
50	−0·33	−0·47	−0·22	−0·34	+0·14	−0·31	−0·17	−0·11
51	0·00	+0·88	+1·80	+0·89	−0·13	−0·10	−0·26	−0·16
52	+0·16	+0·12	+0·02	+0·10	+0·06	−0·05	−0·28	−0·09
53	−0·04	−0·04	−0·11	−0·06	−0·24	+0·46	+0·06	+0·09
54	−0·48	+0·07	−0·11	−0·17	+0·21	+0·54	+0·31	+0·35
Atlas 55	+0·03	+0·13	+0·34	+0·17	+0·14	−0·28	−0·25	−0·13
Pleione 56	−0·14	−0·57	−0·03	−0·25	+0·33	−0·20	+1·09	+0·41
57	−0·10	+0·16	+0·21	+0·09	+0·42	+0·20	+0·26	+0·29
58	−0·05	−0·51	−0·11	−0·22	+0·09	+0·19	+0·19	+0·16
59	−0·12	−0·18	+0·10	−0·07	+0·36	+0·35	−0·34	+0·12
60	−0·70	−0·63	−0·20	−0·51	+0·24	+0·47	−0·06	+0·22
61	+0·46	+0·28	...	+0·37	+0·54	+0·03	...	+0·28
62	−0·05	+0·41	+0·46	+0·27	+0·25	+0·30	−0·15	+0·13
63	+0·12	+0·94	−0·10	+0·32	−0·08	−0·29	−0·46	−0·28
64	+0·15	−0·24	−0·33	−0·14	−0·52	−0·99	−0·98	−0·83
65	+0·70	−0·29	−0·11	+0·10	+0·42	+0·14	−0·21	+0·12
66	−0·41	+1·06	−0·03	+0·21	+0·28	−0·41	−0·05	−0·06

15. Since my conviction that the above process is as valid as it is simple may not be shared by others, it would seem advisable to add a few words in defence of the process.

16. The most striking difference between the above method of reduction and those commonly employed is that in the latter

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various corrections, such as those for refraction and aberration, are first applied to the measures ; that the scale value and orientation of the plate are also separately considered and the proper corrections applied; while in the above reductions all these are (not *neglected*, but) deduced *en bloc* from the measures themselves.

17. Let us consider this difference a little more in detail.

If x and y be the measured coordinates of any star, the refraction correction may be expressed by the formula

$$\delta_1 x = R_1 x + R_2 y,\ \delta_1 y = R_3 x + R_4 y.$$

Various expressions may be given for R_1, R_2, R_3, R_4, but it is only necessary to note that they are constant over the plate, the zenith-distance not being too large.

Similarly the corrections for aberration, &c., may be written

$$\delta_2 x = A_1 x + A_2 y,\ \delta_2 y = A_3 x + A_4 y.$$

The correction for scale value may be written

$$\delta_3 x = Sx,\ \delta_3 y = Sy$$

if we consider the scale value equal in all directions; and that for orientation

$$\delta_4 x = Mx + Ny,\ \delta_4 y = -Nx + My,$$

where

$$M^2 + N^2 = 1.$$

Finally, the correction for error of centring is

$$\delta_5 x = F,\ \delta_5 y = G,$$

assuming that the plate is normal to the axis of the telescope.

Thus the ordinary process of direct correction for such systematic errors would reduce our measures x and y to the following corrected values :

$$\left.\begin{aligned} X &\equiv x + (R_1 + A_1 + S + M)x + (R_2 + A_2 + N)y + F \\ Y &\equiv y + (R_3 + A_3 - N)x + (R_4 + A_4 + S + M)y + G \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

18. If all the quantities $R_1 R_2 \ldots$, $A_1 A_2 \ldots$, S, M, N are considered *completely determined*, then the measures are now finally corrected, and the residuals obtained by converting them into R.A. and N.P.D. and comparing them with known stars must be considered free from systematic error. But though the coefficients $R_1 R_2 \ldots$ and $A_1 A_2 \ldots$ may be calculated with sufficient accuracy, this is generally not the case with S, M, and N. It is very difficult, if not impossible, to determine the scale value of the plate and its orientation *independently of the measures made on the actual stars*. In short, most investigators have hitherto considered that the above corrected measures, X and Y, are affected with small systematic errors $r_1 x + r_2 y + r_3$, $r_4 x + r_5 y + r_6$, the coefficients r_1, r_2, &c., being determined from

the known stars. They have, in fact, solved a series of equations of the form

$$\left.\begin{aligned} r_1x + r_2y + r_3 &= X - \xi \\ r_4x + r_5y + r_6 &= Y - \eta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where ξ and η are, as above, the coordinates of the known stars.

19. Now, once the necessity for this final correction of the plate is admitted, the labour expended in applying the above *direct* corrections as a preliminary must rank as useless. If we form, as above, the differences $x-\xi$, $y-\eta$ between the uncorrected measures x, y and the known coordinates ξ, η and solve the equations

$$\left.\begin{aligned} s_1x + s_2y + s_3 &= x - \xi \\ s_4x + s_5y + s_6 &= y - \eta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

by the same process (of least squares or otherwise) by which we ultimately solve equations (2), we should find

$$\left.\begin{aligned} s_1 &= r_1 - (R_1 + A_1 + S + M) \\ s_2 &= r_2 - (R_2 + A_2 + N) \\ s_3 &= r_3 - F \\ &\text{\&c.} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

that is to say, the total corrections applicable to the measures would be found exactly the same as before. We can see this at once by substituting for X, Y on the right of equations (2) their values from equations (1), thus obtaining

$$r_1x + r_2y + r_3 = x - \xi + (R_1 + A_1 + S + M)x + (R_2 + A_2 + N)y + F$$
$$r_4x + r_5y + r_6 = y - \eta + (R_3 + A_3 - N)x + (R_4 + A_4 + S + M)y + G,$$

which may be rewritten

$$\{r_1 - (R_1 + A_1 + S + M)\}x + \{r_2 - (R_2 + A_2 + N)\}y + r_3 - F = x - \xi$$

and

$$\{r_4 - (R_3 + A_3 - N)\}x + \{r_5 - (R_4 + A_4 + S + M)\}y + r_6 - G = y - \eta,$$

and on comparing these with equations (3), viz.,

$$s_1x + s_2y + s_3 = x - \xi$$
$$s_4x + s_5y + s_6 = y - \eta$$

we see that

$$s_1 = r_1 - (R_1 + A_1 + S + M)$$
$$s_2 = r_2 - (R_2 + A_2 + N)$$
$$\text{\&c. \&c.}$$

Hence, as far as the final correction of the plate is concerned, we get precisely the same result by solving equations (3) at once as by laboriously first correcting the measures and then treating the

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residuals; provided always that we cannot satisfactorily determine the scale value and orientation independently of the measures themselves.

20. It is my own opinion that the scale value and orientation can *not* be determined satisfactorily independently—that is to say, from other plates; and I think this opinion is shared by others who have discussed such measures, especially MM. Loewy and Bakhuyzen.

By "independently" I should understand "from measures made on other plates"; for it is to be remarked that anything attaching to the same plate can be included in the equations (3), with any weight considered advisable. For instance, a star trail which might be considered a determination of orientation independent of the measures is merely a series of stars of the same declination, and will furnish equations to determine s_1, s_2, s_3, &c., which may be included in the set (3). Similarly, if an experiment is made for scale value by impressing two images of the same star on the plate at a known interval, we have practically *two* known stars to be included in equations (3) instead of one.

21. It is to be further remarked that it is quite easy when once the coefficients s_1, s_2, s_3, &c., are obtained to analyse them into their constituent parts r_1, R_1, A_1, S_1M &c., if so desired. The coefficients R_1 R_2 . ., and A_1 A_2 . . must be calculated from the data as to the taking of the plate, and applied to s_1, s_2, &c., so that we find

$$\begin{aligned}
[s_1+R_1+A_1] &= r_1-(S+M)\\
[s_2+R_2+A_2] &= r_2-N\\
[s_3] &= r_3-F\\
[s_4+R_3+A_3] &= r_4+N\\
[s_5+R_4+A_4] &= r_5-(S+M)\\
[s_6] &= r_6-G.
\end{aligned}$$

If we assume $r_1=r_2=r_3=r_4=r_5=r_6=0$, that is, that all the systematic errors are attributable to error of centring, orientation, and an unknown *but uniform* scale value, we have F and G determined; and four equations to find the two quantities S + M and N.

If, however, we admit that the scale value may be different in different directions, we cannot separate the scale value from the orientation, and some of the small quantities r_1, r_2, r_4, r_5 must be retained. If the quantities in square brackets are found to be the same for different plates, we may assume that the scale value and orientation have remained sensibly constant.

22. The values of the coefficients in Table III. are deduced from a few selected stars only, which may be affected with large accidental errors or with large proper motions; and hence the resulting formulæ may not be the best for the whole plate. But it is a well-known characteristic of linear equations that one

solution may be readily added to another, and hence we can improve our solution by treatment of the residuals given in Table IV. to any extent we choose.

For example, let us divide the stars *not* used in the above solution into four groups as follows—

Group V.: $(x-, y+)$; Nos. 10, 14, 17, 18, 19, 20, 22, 23, 24, 25, 27, 33, 35

Group VI.: $(x-, y-)$; Nos. 8, 26, 28, 29, 32, 36

Group VII.: $(x+, y-)$; Nos. 39, 42, 51, 54, 57, 62, 66

Group VIII.: $(x+, y+)$; Nos. 37, 38, 44, 49, 50, 52, 53, 58, 59, 61, 65

and form the mean coordinates and residuals for these groups, we find

TABLE V.

Group.	Mean ξ.	Mean η.	Mean x Residual. Plate 252.	Plate 259.	Plate 294.	Mean Plate.	Mean y Residual. Plate 252.	Plate 259.	Plate 294.	Mean Plate.
	′	′	″	″	″	″	″	″	″	″
V.	−19·428	+15·125	−0·01	−0·23	−0·09	−0·11	−0·37	+0·04	−0·02	−0·12
VI.	−13·358	−15·408	+0·17	+0·21	−0·02	+0·12	+0·13	+0·29	+0·19	+0·20
VII.	+16·957	−19·453	−0·21	+0·41	+0·25	+0·15	+0·14	0·00	+0·14	+0·09
VIII.	+14·054	+11·769	+0·18	−0·15	+0·11	+0·05	+0·13	+0·11	−0·07	+0·06

23. Let us now assume that these residuals can be corrected by an expression of the form $\gamma + \alpha\xi + \beta\eta$: then for the x residuals we have 4 equations (one for each group), to determine α and β for each plate. The equations from the first two groups are

$$\gamma - 19{\cdot}428\ \alpha + 15{\cdot}125\ \beta = -0''{\cdot}01,\ -0''{\cdot}23,\ \text{or}\ -0''{\cdot}09\ \text{respectively,}$$

and

$$\gamma - 13{\cdot}358\ \alpha - 15{\cdot}408\ \beta = +0''{\cdot}17,\ +0''{\cdot}21,\ \text{or}\ -0''{\cdot}02\ \text{respectively.}$$

Subtracting to eliminate γ we have

$$-6{\cdot}1\ \alpha + 30{\cdot}5\ \beta = -0''{\cdot}18,\ -0''{\cdot}44,\ \text{or}\ -0''{\cdot}07\ \text{respectively.}$$

Similarly from Groups VII. and VIII. we get

$$-2{\cdot}9\ \alpha + 31{\cdot}1\ \beta = +0''{\cdot}39,\ -0''{\cdot}56,\ \text{or}\ -0''{\cdot}14\ \text{respectively.}$$

Now the coefficients of these two equations are so nearly similar that, unless β is very small and α large, the quantities on the right ought to agree well if they denote real systematic errors. A difference such as that between $-{}''{\cdot}18$ and $+{}''{\cdot}39$ must mean that the errors are largely accidental.

Similarly, forming equations VIII.–V., and VII.–VI., we get

$$+33{\cdot}5\ \alpha - 3{\cdot}4\ \beta = +0''{\cdot}19,\ +0''{\cdot}08,\ \text{or}\ +0''{\cdot}20$$
$$+30{\cdot}3\ \alpha - 4{\cdot}0\ \beta = -0''{\cdot}38,\ +0''{\cdot}20,\ \text{or}\ +0''{\cdot}27.$$

And if we take the y residuals we get the following six pairs of quantities, which should, but do not generally agree:

$-0''\!.50$, $-0''\!.25$, or $-0''\!.21$ (V.–VI.)	$+0''\!.50$, $+0''\!.07$, or $-0''\!.05$ (VIII.–V.)
-0.01, $+0.11$, or -0.21 (VIII.–VII.)	$+0.01$, -0.29, or -0.05 (VII.–VI.)

It seems almost unnecessary, therefore, to solve these equations, as it would appear that the limit of accuracy has been nearly reached.

24. It must be remembered that the errors of the *réseau*, the tilt of the plate, and the optical distortion have been neglected. There are also several other directions in which the measures under discussion might be improved which it is not necessary to discuss here. The present investigation is not intended to show the limit of accuracy which may be obtained from a photograph; rather is it intended to point out that, with labour which (in my opinion) considerably exceeds that which can be afforded for the measurement of a plate for the Astrographic Chart, we must be content to accept accidental errors of at least $\pm 0''\!\cdot\!25$. How far these errors may be reduced by refined investigations is a question of the utmost importance, but one which those concerned with the completion of the Astrographic Chart must (in my opinion) resolutely put aside for the present.

25. On the other hand, I consider that it is shown by the above and similar discussions how very rapidly stellar positions may be obtained from the plates with an accuracy of about $\pm 0''\!\cdot\!50$. After all, this does not compare unfavourably with the accuracy of the meridian observations which must form our basis of operations; and in the work for the Chart it is important to remember that the saving of a single figure for each star reduces the labour appreciably.

26. It is in favour of the work on the Chart plates that this investigation is here terminated for the present.

Photograph of the Cluster ℍ VI. 37 *Argûs.*

By Isaac Roberts, D.Sc., F.R.S.

The photograph of the cluster ℍ VI. 37 *Argûs*, R.A. $7^h\ 55^m$, Decl. $10°\ 20'$ south, was taken with the 20-inch reflector on 1894 February 27, with exposure of the plate during 90 minutes, and the copy now presented is enlarged to the scale of 1 millimetre to 24 seconds of arc.

The cluster is No. 2,506 in the *New General Catalogue* and No. 1,611 in the *General Catalogue*, where (p. 75) Sir J. Herschel describes it as pretty large, very rich, compressed, stars 11–20 mag.